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Reggeon Scattering in an External Field: A Solitonic Model

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Solitonic solutions of the nonlinear Schrödinger equation are considered to represent reggeons. The effect of a constant electric field on reggeons is formulated and the modified energy levels due to the interaction are calculated using the usual semiclassical approach. In summing overall stability angles a simple subtraction procedure is adopted for regularization.

1. INTRODUCTION

Modern physics has so far developed primarily on the basis of the recognition of linear characteristics of natural phenomena. Various types of interactions have been treated by perturbative approaches. But this kind of decomposition of physical processes involves complications, and in fact the perturbative approach often has serious difficulties, such as divergent results in each order of perturbation and a lack of convergence of the perturbative series. In nature, the success of a linear theory is rather exceptional and in most cases the nonlinearity plays an essential role. From this point of view we identify the solitonic modes of the nonlinear Schrödinger equation (NLSE) (Girardello and Jengo, 1977) with reggeons, against the perturbative approach. The colored modes of reggeons are discussed in Roy Chowdhury and Sidhanta (1986).

The present work studies the effect of a constant electric field on the solitonic modes of reggeons. The semiclassical path integral formalism has already been applied by Dashen *et al.* (1975; also see deVega and Matllet, 1982) and its utility as useful tool is quite established.

The major motivation of the paper is to find the effect of an electric field upon the energy levels, as in the Stark effect in atomic physics. The

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renormalization problem is not dwelt upon in this discussion, since it has been thoroughly discussed elsewhere. But we find that the sum over all stability angles shows renormalizability by a simple subtraction procedure analogous to the BPHZ formalism.

2. PRELUDE

The nonlinear Schrödinger equation can also be derived by the inverse scattering method. This method is different from that of Lax, but the results are equivalent. The method of Zakharov and Shabat (1973) is to consider a modification of the scattering problem, which starts with the coupled equations

$$\begin{aligned} \vartheta_{1x} &= -i\rho\vartheta_1 + q(x, t)\vartheta_2 \\ \vartheta_{2x} &= i\rho\vartheta_2 + \gamma(x, t)\vartheta_1 \end{aligned} \tag{1}$$

and the most general linear time dependence which is local

$$\begin{aligned} \vartheta_{1t} &= A\vartheta_1 + B\vartheta_2 \\ \vartheta_{2t} &= C\vartheta_1 + D\vartheta_2 \end{aligned} \tag{2}$$

With the choice

$$\begin{aligned} A &= 2i\rho^2 \pm iqq^* \\ B &= 2q\rho + iq_x \\ C &= \mp 2q^*\rho \pm iq^*x \end{aligned} \tag{3}$$

we have the NLSE

$$iq_t = q_{xx} \pm 2q^2 q^* \tag{4}$$

For brevity we start with the two-soliton solution of the NLSE with vanishing asymptotic condition as given by

$$\phi^{(2)} = \sum \lambda_k^*(1 - A)_{lk}^{-1} \lambda_k^* \tag{5}$$

where

$$A = \begin{pmatrix} c_{11}^2 + c_{21}^2 & c_{11}c_{12} + c_{21}c_{22} \\ c_{12}c_{11} + c_{22}c_{21} & c_{12}^2 + c_{22}^2 \end{pmatrix} \tag{6}$$

and

$$\begin{aligned} c_{jk} &= \tilde{\lambda}_j \tilde{\lambda}_k / \lambda_j - \lambda_k^* \\ \tilde{\lambda}_1 &= (2\eta_1)^{1/2} \exp \chi_1 \\ \tilde{\lambda}_2 &= (2\eta_2)^{1/2} \exp \chi_2 \end{aligned} \tag{7}$$

with

$$\chi_j = 2i(\xi_j^2 - \eta_j^2)t + 2\xi_j x - 4\xi_j\eta_j t - \eta_j x \tag{8}$$

The one-soliton solution appears to be

$$\phi^{(1)} = \tilde{\lambda}_1^*\tilde{\lambda}_1^*/[1-(\tilde{\lambda}_1\tilde{\lambda}_1^*/\lambda_1 - \lambda_1^*)^2] \tag{9}$$

Expanding in the neighborhood of the one-soliton solution, we calculate the fluctuation to be

$$\delta\phi = -\varepsilon\left[\frac{A}{D} - \frac{\lambda_1\lambda_1^*(1-C_{22}^2)B}{D^2}\right] \tag{10}$$

when

$$D = (1-c_{22}^2)(1-c_{11}^2)$$

$$A = 2\sigma_1^2\eta_1 \exp 2x_1^* - 4(\sigma_1 c_{11} + \sigma_2 c_{22}) \exp(x_1 + x_1^*) + 2\exp 2x_2^*(1-c_{11}^2)$$

$$B = \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2 c_{11}c_{12}$$

$$C_{11} = \lambda_1\lambda_1^*/2\eta_1^2 = -i\exp(-8\xi_1\eta_1 t - 2\eta_1 x)$$

$$C_{22} = \lambda_2\lambda_2^*/2\eta_2^2 = -i\exp(-8\xi_2\eta_2 t - 2\eta_2 x), \text{ putting } \eta_2 = \varepsilon \text{ and } \xi_2 = K$$

Imposing a periodicity condition in a box of length L upon the fluctuation, we have

$$2(\xi_1 - K_n)L + \tan^{-1}\frac{\xi_1^2 - \eta_1^2 + K_n^2 - 2\xi_1 K_n}{2\eta_1 - K_n)} = 2\pi n \tag{11}$$

where $n = 0, 1, 2, \ldots$ corresponds to the periodic paths. The quantized energy of stability angles are defined to be

$$E = 4(\xi_1^2 - \eta_1^2 - K_\lambda^2) \tag{12}$$

In the absence of an electric field the usual Lagrangian is always written in the case of the ϕ^2 interaction in the form

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\frac{\partial}{\partial t}\psi - \alpha\,\nabla\bar{\psi}\,\nabla\psi - \Delta_0\bar{\psi}\psi + g\psi\bar{\psi}\psi\bar{\psi} \tag{13}$$

and this yields the equation of motion in the form

$$i\,\partial\psi/\partial t - \alpha\,\nabla^2\psi - \Delta_0\psi + g\psi\bar{\psi}\psi = 0 \tag{14}$$

In order to drop the explicit occurrence of the masslike term, we use the shifted field

$$\psi = \phi\, e^{iA_0 t} \tag{15}$$

We consider the case of a pomeron where $\alpha = 1$, and the soliton solution is taken in the form

$$\phi^{1S} = \eta_1 \left(\frac{8}{g}\right)^{1/2} \exp[-4i(\xi_1^2 - \eta_1^2)t - 2i\xi_1 x] \operatorname{sech}(8\xi_1\eta_1 t + 2\eta_1 x) \quad (16)$$

3. INTERACTION WITH THE ELECTRIC FIELD

In the presence of a constant electric field the equation of motion is taken in the form

$$i\psi_t + \psi_{xx} = g\psi^+\psi\psi - eEx\psi \quad (17)$$

The solution of the NLS equation in the absence of an electric field E is represented by (16) and the solution to (17) is related by the transformation equation as

$$\psi(t, x + eEt^2) = \phi(t, x) \exp[iEtx + \tfrac{2}{3}(eE)^2 t^3] \quad (18)$$

Hence, the solution to equation (17) is given by

$$\begin{aligned}\psi(x, t) = \eta_1 \left(\frac{8}{g}\right)^{1/2} &\exp[-4i(\xi_1^2 - \eta_1^2)t - 2i\xi_1(x - eEt^2)] \\ &\times \operatorname{sech}[8\xi_1\eta_1 t + 2\eta_1(x - eEt^2)] \\ &\times \exp i[eEt(x - eEt^2) + \tfrac{2}{3}(eE)^2 t^3]\end{aligned} \quad (19)$$

The fluctuation of the two-soliton solution about the one-soliton solution is calculated on the basic of expression (10), but for that we have to redefine certain functions as given by

$$\tilde{\alpha}_1 = K_1^{1/2} e^{Y_1}, \qquad \tilde{\alpha}_2 = K_1^{1/2} e^{Y_2} \quad (20)$$

where

$$\begin{aligned} Y_i &= 2i(\xi_i^2 - \eta_i^2)t + i\xi_i(x - eEt^2) - 4\xi_i\eta_i t - \eta_i(x - eEt^2) \\ K_1 &= \eta_1 \left(\frac{8}{g}\right)^{1/2} \exp i[eEt(x - eEt^2) + \tfrac{2}{3}(eE)^2 t^3] \end{aligned} \quad (21)$$

The one-soliton solution in the presence of a constant electric field is given by

$$\Psi^{(1)} = \tilde{\alpha}_1^* \tilde{\alpha}_1^* \Big/ 1 - \left(\frac{\tilde{\alpha}_1 \tilde{\alpha}_1^*}{\alpha_1 - \alpha_1^*}\right)^2 \quad (22)$$

The fluctuation of the two-soliton solution is calculated by collecting the important terms at large distance:

$$d\phi = K_1^2 \exp(Y_1 + Y_1^*) \left[\frac{\exp(2Y_1 + 2Y_2^*)}{(\xi_1 + i\eta_1 - K)^2} + \frac{\exp(2Y_2 + 2Y_1^*)}{(\xi_1 - i\eta_1 - K)^2} \right] \tag{23}$$

Thus

$$\begin{aligned} d\phi = \eta_1^2 \left(\frac{8}{g}\right) & \exp i[2eEt(x - eEt^2) + \tfrac{4}{3}(eE)^2 t^3] \\ & \times \exp[-16\xi_1\eta_1 t - 4\eta_1(x - eEt^2)] \\ & \times \left[\frac{e^{iu}}{(\xi_1 + i\eta_1 - K)^2} + \frac{e^{-iu}}{(\xi_1 - i\eta_1 - K)^2} \right] \end{aligned} \tag{24}$$

where

$$u = \left[4(\xi_1^2 - \eta_1^2 - K) + 2(K - \xi_1) \frac{eE\varepsilon}{\omega} \right] t + 2(\xi_1 - K)x \tag{25}$$

Equation (24) can be put into the form

$$\begin{aligned} d\phi = \eta_1^2 \left(\frac{8}{g}\right) & \exp(ip_1) \exp(p_2) \\ & \times \sin\left\{ \left[4(\xi_1^2 - \eta_1^2 - K^2) + 2(K - \xi_1) \frac{eE\varepsilon}{\omega} \right] t + 2(\xi_1 - K)x \right. \\ & \left. + \tan^{-1} \frac{\xi_1^2 - \eta_1^2 + K_n^2 - 2\xi_1 K_n}{2\eta_1(\xi_1 - K_n)} \right\} \end{aligned} \tag{26}$$

where

$$p_1 = 2eEt(x - eEt^2) + \tfrac{4}{3}(eE)^2 t^3$$

$$p_2 = -16\xi_1\eta_1 t - 4\eta_1(x - eEt^2)$$

Imposition of the periodicity condition in a box of length L yields a condition exactly the same as given by (11).

The corresponding quantized energy levels are given by

$$\mathscr{E}_n = \frac{4(\xi_1^2 - \eta_1^2 - K_n^2)}{1 - 2(\xi_1 - K_n)eE/\omega} \tag{27}$$

The expression shows the effect of an electric field upon the energy levels of the reggeons.

4. DISCUSSION

The semiclassical path integral formalism is not always immune to the divergence problem, as pointed out by Dashen *et al.* (1975) and deVega and Matllet (1982) and others in the context of sine-Gordon field theory. Our experience is similar with the NLSE in the presence of an external electric field. We subtract the vacuum contribution and sum over the stability angles and define

$$\xi-\xi_0=-\tfrac{1}{2}\sum \nu+\tfrac{1}{2}\sum \nu_0 \tag{28}$$

This yields

$$\xi-\xi_0=-\frac{\tau}{2}\left[\sum\frac{4(\xi_1^2-\eta_1^2-K_n^2)}{1-(\xi_1-K_n)eE/\omega}-\sum\frac{4[\xi_1^2-\eta_1^2-(2\pi n/L)^2]}{1-[\xi_1-(2\pi n/L)^2]eE/\omega}\right] \tag{29}$$

It is nontrivial to recast this into the form

$$\xi-\xi_0=-\frac{\tau}{2}\left[\frac{1}{2\pi}\int_0^\infty\frac{\partial}{\partial K}\left\{\frac{4(\xi_1^2-\eta_1^2-K^2)}{1-(\xi_1-K)eE/\omega}\right\}\right]\delta\, dK \tag{30}$$

where

$$\delta=\tan^{-1}\frac{\xi_1^2-\eta_1^2+K^2-2\xi_1K}{2\eta_1(\xi_1-K)} \tag{31}$$

For large K we subtract $(\tau/\pi\alpha)\tan^{-1}(K/2\eta_1)$ from equation (31).

The redefined sum of stability angles yields

$$(\xi-\xi_0)_{\mathrm{R}}=\frac{\tau a}{2\alpha^2}\frac{2\alpha\xi_1-\alpha^2\eta_1^2-1}{4\alpha^2\eta_1^2+1+\alpha^2\xi_1^2-2\alpha\xi_1} \tag{32}$$

where $\alpha=eE/\omega$ and $a=1-\alpha\xi$. Hence the effective action can be written as

$$S_{\mathrm{eff}}=S_{\mathrm{cl}}+\frac{\tau a}{2\alpha^2}\frac{2\alpha\xi_1-\alpha^2\eta_1^2-1}{4\alpha^2\eta_1^2+1+\alpha^2\xi^2-2\alpha\xi_1} \tag{33}$$

At this stage the limit $E=0$ (i.e., the free field limit) should not be sought in (32); on the contrary, the limit $E=0$ should be taken in the expression for the stability angle before considering the limit $K\to\infty$ and the regularization is performed. With this procedure the free field limit can always be reproduced.

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